☺ 1.1 – Recursively Defined Sequences ☺

Objectives:

1. Discover recursive formulas for sequences
2. Define, explore, and use of arithmetic and geometric sequences
3. Use recursively defined sequences to model real-life situations

Arithmetic Sequence:

Sequence in which each term is equal to the previous term plus a constant (*common difference*)

u_1 = *starting number*

$u_n = u_{n-1} + d$ (where d is the common difference)

Example 1: Each square in this pattern has side length 1 unit. Imagine that the pattern continues.

10 16 22 28

34 40 46 52 58 64 70

5 6 7 8 9 10 11

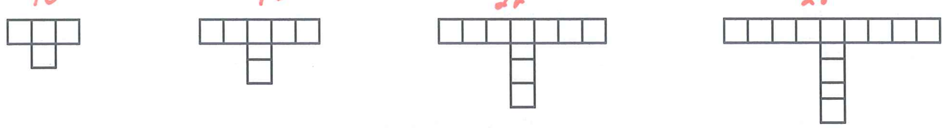


Figure 1 Figure 2 Figure 3 Figure 4

a. Find the perimeter of Figure 9. 58

b. Which figure has a perimeter of 76 units? 12

c. Write a recursive definition to find the perimeter of any figure in the pattern.

$u_n = u_{n-1} + 6$ where $n \geq 2$

$u_1 = 10$

Sequence: Ordered list of numbers.

Term: Each number in the sequence

General Term: The n^th term → a term at an unknown place in the sequence

Recursive Formula: Formula that defines a sequence ~ 1) starting term(s) (initial value)

2) recursive rule

Recursive Rule: Defines the n^th term in relation to the previous term

Example 2: A concert hall has 59 seats in row 1, 63 seats in row 2, 67 seats in row 3, and so on. The concert hall has 35 rows of seats.

a. Write a recursive formula to find the number of seats in each row.

$u_1 = 59$

$u_n = u_{n-1} + 4$ where $n \geq 2$

b. How many seats are in row 4?

$u_2 = u_1 + 4$ $\quad u_3 = u_2 + 4$ $\quad u_4 = u_3 + 4$

$u_2 = 63$ $\quad u_3 = 67$ $\quad \boxed{u_4 = 71}$

c. Which row has 95 seats?

$u_5 = 75$ $\quad u_6 = 79$ $\quad u_7 = 83$ $\quad u_8 = 87$ $\quad u_9 = 91$ $\quad u_{10} = 95$

$\boxed{\text{Row } 10}$

Example 3: Write a recursive formula that you can use to find the number of segments, u_n, for Figure n of this arithmetic pattern. Use your formula to complete the table.

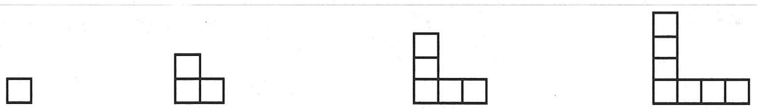


Figure 1 Figure 2 Figure 3 Figure 4

Recursive Routine: $u_1 = 4$

$u_n = u_{n-1} + 6$ where $n \geq 2$

$10 \times 6 = 60 + 10 = 70$

Figure	1	2	3	4	5	…	12	…	32
Segments	4	10	16	22	28	…	70		190

$120 \div 6 = 20 + 12 = 32$

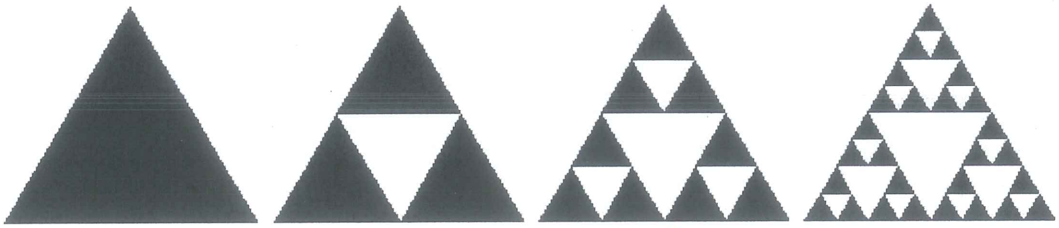
Geometric Sequence:

Sequence in which each term is equal to the previous term multiplied by a constant (*common ratio*)

u_1 = <u>*starting number*</u>

$u_n = r \cdot u_{n-1}$ (where r is the common ratio)

Example 4: The geometric pattern below is created recursively. If you continue the pattern endlessly, you create a **fractal** called the Sierpinski triangle.



1 3 9 27

a. Write a recursive formula to represent that number of *black* triangles.

$u_1 = 1$

$u_n = 3u_{n-1}$ where $n \geq 2$

b. How many *black* triangles are there at stage 20?

1,162,261,467 black triangles

Example 5: Use the sequence below to answer the following questions:

9, 13.5, 20.25, 30.375, …

a. Is this sequence geometric or arithmetic? How do you know??

GEOMETRIC - MULTIPLY BY 1.5 EACH TIME

b. Write the recursive routine for this sequence:

$u_1 = 9$

$u_n = u_{n-1} \times 1.5$

How do the recursive formulas for arithmetic and geometric sequences differ?

Arithmetic are created by adding the same number each time while geometric are created by multiplying by the same number each time.